



Pioneering Work on Survey of Parallel Nonlinear System Solvers

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In the early days of parallel computing, researchers were exploring ways to solve complex mathematical problems more efficiently by harnessing the power of multiple processors. One influential work from this era was the 1991 paper “A Survey of Parallel Nonlinear System Solvers” by John Ortega and Robert Foigel.

At the time, many scientific and engineering applications required solving large systems of nonlinear equations, which could be extremely computationally intensive. Ortega and Foigel reviewed the state-of-the-art in parallelizing classical iterative methods like the Gauss-Jacobi method for solving such systems on parallel architectures.

The Gauss-Jacobi method is an iterative technique for solving a system of n nonlinear equations in n unknowns. Each iteration updates the solution approximations in parallel by solving each equation for its corresponding unknown, using the previous iteration’s approximations for all other unknowns.

Ortega and Foigel examined how this inherently parallel algorithm could be effectively implemented on parallel computing systems available in the early 1990s. They analyzed its convergence properties, synchronization requirements between processors, and potential performance gains over sequential implementations.

A key focus was on asynchronous parallel implementations, where processors could update solution approximations as soon as operand data was available, rather than synchronizing after each iteration. This offered potential speedup over synchronous implementations on parallel systems with many processors.

The paper provided a comprehensive survey of the existing work in this area up to 1991. It highlighted successful parallel implementations of the Gauss-Jacobi and related methods on parallel architectures like the Hypercube, Butterfly, and Intel iPSC systems that were cutting-edge at the time.

While the specific hardware platforms are now obsolete, the insights around parallelizing iterative solvers have remained highly relevant as the number of processors/cores in modern systems has exponentially increased. Ortega and Foigel’s work helped lay the foundations for effectively utilizing parallel computing power for demanding numerical problems.

Their paper was a pioneering effort in understanding how to map classical numerical algorithms onto the parallel computing architectures emerging in the late 20th century. It exemplified the mathematical analysis and practical implementation considerations required to fully harness the potential of parallel systems.

In 1991, John Ortega and Robert Foigel published a groundbreaking article titled “A Survey of Parallel Nonlinear System Solvers.” The paper aimed to provide an in-depth examination of parallel implementations of the Gauss-Jacobi method, a popular iterative technique used for solving systems of nonlinear equations. This article not only highlighted the potential of parallel computing in accelerating the solution process but also laid the foundation for future research in parallel nonlinear system solvers.

The Gauss-Jacobi method, named after Carl Friedrich Gauss and Carl Gustav Jacob Jacobi, is a well-established technique for solving systems of linear equations. It involves updating each variable in the system simultaneously, using the current values of all other variables. Despite its simplicity, the method can be extended to handle nonlinear systems, albeit with additional complexity. The parallelization of this method becomes particularly interesting when dealing with large-scale problems, as it allows for distributing the computational load across multiple processors, thereby significantly reducing the overall computation time.

Ortega and Foigel’s survey delved into various aspects of parallelizing the Gauss-Jacobi method. They began by discussing the fundamental concepts of parallel computing, emphasizing the need for efficient communication and synchronization mechanisms between processors. The authors highlighted that parallelizing iterative methods like Gauss-Jacobi requires careful consideration of load balancing, data dependencies, and communication overheads, which can significantly impact the overall performance.

One of the key contributions of the article was the review of different parallelization strategies for the Gauss-Jacobi method. They classified these strategies into three categories: data decomposition, domain decomposition, and a combination of both. Data decomposition involves distributing the system variables among processors, allowing each processor to independently update a subset of the variables. Domain decomposition, on the other hand, partitions the problem domain into smaller subdomains, with each processor responsible for solving the equations within its assigned subdomain. The combination of both techniques combines the advantages of data and domain decomposition, achieving better load balancing and communication efficiency.

The authors also discussed various synchronization techniques employed in parallel Gauss-Jacobi implementations. These included centralized, decentralized, and hybrid approaches, each with its own trade-offs in terms of communication overhead and convergence rate. They analyzed the impact of these synchronization strategies on the overall performance, providing insights into the optimal choices for different problem sizes and system architectures.

In addition to theoretical discussions, Ortega and Foigel presented a comprehensive survey of existing parallel implementations of the Gauss-Jacobi method. They reviewed parallel algorithms proposed by researchers in academia and industry, comparing their performance and highlighting the challenges faced in their implementation. This section provided practical insights into the real-world application of parallel nonlinear system solvers and served as a valuable reference for researchers and practitioners.

The article concluded by identifying future research directions and challenges in parallel nonlinear system solvers. They emphasized the need for more sophisticated load balancing algorithms, dynamic scheduling techniques, and adaptive synchronization strategies to further enhance the efficiency of parallel Gauss-Jacobi methods. The authors also highlighted the importance of considering the specifics of the underlying hardware, such as communication networks and processor architectures, when designing parallel algorithms.

In summary, "A Survey of Parallel Nonlinear System Solvers" by John Ortega and Robert Foigel was a seminal work that provided a comprehensive overview of parallel implementations of the Gauss-Jacobi method. The article not only documented the state-of-the-art techniques but also offered valuable insights into the challenges and future directions in the field. It served as a benchmark for researchers and practitioners, inspiring further advancements in parallel nonlinear system solvers and contributing to the ongoing quest for more efficient and scalable algorithms in scientific computing.

[← "Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods" \(1994\) by Richard Barrett et al.](#)
